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AN ITERATIVE GUIDANCE SCHEME FOR ASCENT TO ORBIT
(SUBORBITAL START OF THE THIRD STAGE)

Isaac E. Smith and Emsley T. Deaton, Jr. 29 May 1963 39p refs

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GUIDANCE THEORY SECTION
DYNAMICS ANALYSIS BRANCH
AEROBALLISTICS DIVISION

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ABSTRACT

Average gravity magnitude and direction approximations allow a closed form solution to the equations of motion. The solution yields a time-to-go before cutoff as well as a steering function, consisting of a thrust attitude and a thrust attitude turning rate. The principles of the guidance scheme, which is adaptive for a set of large disturbances, are outlined with a constant gravity. A spherical earth model extension, which includes staging during the guidance phase, is presented. Performance data and a comparison of trajectory shaping are included where the comparison is against the theoretical optimum classical calculus of variations solution.

AUTHOR

DEFINITION OF SYMBOLS

Symbol	Definition
$\ddot{x}$, $\ddot{y}$	acceleration components in the space fixed x-y system
$\dot{x}$, $\dot{y}$	velocity components in the space fixed x-y system
x, y	position coordinates in the space fixed x-y system
$\dot{\xi}$, $\dot{\eta}$	velocity components in the injection coordinate system
ξ, η	position coordinates in the injection coordinate system
$\tilde{\chi}_{\xi}$	constant thrust attitude angle reference to the ξ-axis required to meet desired velocity end conditions
χ	thrust attitude angle reference to the x-y axis in the space fixed x-y system
t_1	instantaneous time
T_1, T_2	remaining flight time for stage one and stage two, respectively
R_o	radius of the earth
R_1	distance from the center of the earth to the vehicle
R_T	distance from the center of the earth to the injection point
K_1, K_2	coefficients in the steering function
m_{11}, m_{12}	instantaneous mass of the first and second stage, respectively
F_1, F_2	thrust of the first and second stage, respectively
$\dot{m}_1$, $\dot{m}_2$	mass flow rate of the first and second stage, respectively
$\tau_1 = m_{11}/\dot{m}_1$	complete burn-up time of first stage

TABLE OF CONTENTS

DEFINITION OF SYMBOLS (Cont'd)

Symbol	Definition
$\tau_2 = m_{12}/\dot{m}_2$	complete burn-up time of second stage
$V_{ex_1} = (I_{SP_1})g_o$	characteristic velocity, first stage
$V_{ex_2} = (I_{SP_2})g_o$	characteristic velocity, second stage
g^*	average gravity magnitude between the instantaneous point and the cutoff point
ϕ_T	total range angle
ϕ^*	average g^* direction
T_2^*	remaining second stage burn time computed from the characteristic velocity equation
ΔV_1	instantaneous velocity deficiency
$\Delta\dot{\xi}_1$	instantaneous velocity deficiency in the ξ-direction
$\Delta\dot{\eta}_1$	instantaneous velocity deficiency in the η direction
V	total velocity
δT_2	correction function for T_2
$\dot{\xi}_T$, $\dot{\eta}_T$	nominal or desired cutoff velocity components in the ξ-η system
η_T	desired η component at cutoff
i	subscript denoting inertial values
1	subscript denoting instantaneous values
1, 2	when second subscript is used, denotes first and second stage values, respectively
T	subscript denotes terminal values.

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AN ITERATIVE GUIDANCE SCHEME FOR ASCENT TO ORBIT (SUBORBITAL STAGE OF THE THIRD STAGE)

By

Isaac E. Smith and Emsley T. Deaton, Jr.

SUMMARY

An approximate closed form solution of the equations of motion allows the derivation of a path adaptive guidance scheme for vehicle flight in a vacuum. The scheme is characterized by a limited number of presettings and in-flight computation of the guidance parameters. The generation of some transcendental functions is required; however, no successive approximation procedures are necessary. The guidance outputs are time-to-go before cutoff and the steering function which consists of a thrust attitude and a thrust attitude turning rate, K_2. The turning rate is used for proper attitude control between passes through the guidance computer. Since the booster phase of the trajectories presented are unguided*, perturbations were allowed to build up. It was found that the scheme is adaptive for a large set of first stage disturbances. The adaptive nature of the scheme also allows it to handle second and third stage performance perturbations with only a relatively small loss in injection weights when compared to the calculus of variation trajectories. No control over the ground range from launch to the injection point was attempted.

SECTION I. INTRODUCTION

The proposed guidance scheme is based on a steering program derived from a set of simplified differential equations of motion. The simplification is justified since the implementation of the scheme is basically null seeking. The steering program itself has the calculus of variations as a background.

*Position and velocity information is not explicitly used for steering.

It is well known from literature [1] that, for a flat earth, the optimum thrust attitude is given by

$$\tan(\chi) = \frac{a' + b't}{c' + d't} . \qquad \text{Law No. 1}$$

Imposition of orbital conditions without range control gives

$$\tan(\chi) = a'' + b''t. \qquad \text{Law No. 2}$$

First order expansion of Law No. 1 or 2 yields the form

$$\chi = a + bt. \qquad \text{Law No. 3}$$

A comparison survey of spherical earth trajectories [2] using a calculus of variations procedure and trajectories using Law No. 3 has shown that there is little difference in performance. The guidance scheme presented in this report uses Law No. 3 as a steering program which is updated after each guidance cycle from the state variables that can be made available at that time.

SECTION II. DESCRIPTION

The principles of the scheme can best be demonstrated by assuming a vacuum flat earth with constant gravity, g, as a model. Later in the report extensions will be made to a spherical earth.

The differential equations of motion relative to a vacuum flat earth can be written as

$$\ddot{x} = a \cos \chi \tag{1}$$

$$\ddot{y} = a \sin \chi - g, \tag{2}$$

where a represents the thrust acceleration and χ is the thrust attitude angle referenced to the x-axis. The symbol, a, is the force of the thrust divided by the instantaneous mass of the vehicle. Let

m_1 = instantaneous mass of the vehicle,

$\dot{m}$ = mass flow rate,

F = force of the thrust,

then,

$$a = F/m_1$$

or

$$a = \frac{V_{ex}}{\tau - t} \tag{3}$$

where

$$\tau = m_1/\dot{m} \tag{4}$$

and

$$V_{ex} = g_o\ I_{SP} = F/\dot{m}. \tag{5}$$

The following integrals are given for future use:

$$\int_o^T a\ dt = V_{ex}\ \ln\left(\frac{\tau}{\tau - T}\right)$$

$$\iint_o^T a\ dt^2 = -V_{ex}\left[(\tau - T)\ \ln\left(\frac{\tau}{\tau - T}\right) - T\right]$$

$$\int_o^T a\ tdt = V_{ex}\left[\tau\ \ln\left(\frac{\tau}{\tau - T}\right) - T\right] \tag{6}$$

$$\iint_o^T a\ tdt^2 = V_{ex}\left\{-\frac{T^2}{2} - \tau\left[(\tau - T)\ \ln\left(\frac{\tau}{\tau - T}\right) - T\right]\right\}$$

where T is the time-to-go from any arbitrary instant.

Now, to impose velocity end conditions only, it is well known that, for a flat earth with a constant gravity, a constant thrust direction is sufficient [1], (Fig. 1).

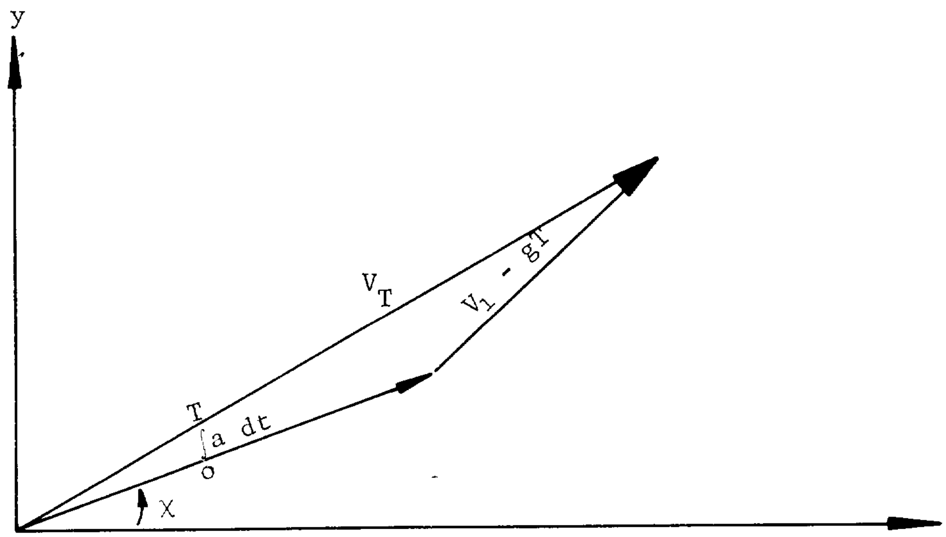

FIGURE 1

Let $\chi = \tilde{\chi}$, a constant, then the first integrals of equations (1) and (2) are

$$\dot{\tilde{x}}_T = \dot{x}_1 + V_{ex} \ln\left(\frac{\tau}{\tau - T}\right) \cos\tilde{\chi}$$

$$\dot{\tilde{y}}_T = \dot{y}_1 + V_{ex} \ln\left(\frac{\tau}{\tau - T}\right) \sin\tilde{\chi} - gT. \tag{7}$$

Equations (7) can be solved for $\tilde{\chi}$; therefore,

$$\tilde{\chi} = \arctan\left[\frac{\dot{\tilde{y}}_T - \dot{y}_1 + gT}{\dot{\tilde{x}}_T - \dot{x}_1}\right]. \tag{8}$$

The deficiency in velocity required to achieve the desired velocity end conditions may be written as

$$V_i = \left[(\dot{\tilde{x}}_T - \dot{x}_1)^2 + (\dot{\tilde{y}}_T - \dot{y}_1 + gT)^2 \right]^{1/2} . \tag{9}$$

The time-to-go, T, may be determined from equation (9) and the characteristic velocity equation

$$\Delta V_i = V_{ex} \ln \left(\frac{\tau}{\tau - T} \right).$$

Hence,

$$T = \tau \left[1 - e^{-\Delta V_i / V_{ex}} \right]. \tag{10}$$

Equations (8), (9) and (10) can be solved for T and $\tilde{\chi}$ for the current state variables $\dot{x}_1$ and $\dot{y}_1$ and the required terminal velocity components $\dot{\tilde{x}}_T$ and $\dot{\tilde{y}}_T$. At this time, assume that T is found from equations (9) and (10) through successive approximation methods. Later a method will be introduced that eliminates any need for iteration processes within the scheme. Since the state variables are continually changing, the determination of T and the computation of $\tilde{\chi}$ proceed stepwise using the up-dated values of $\dot{x}_1$ and $\dot{y}_1$ as they are obtained. As ΔV_i and T approach zero near the end of the powered flight, equation (8) becomes indeterminate; however, it will be shown later that this difficulty is not serious and can be resolved without undue complication.

Now to enforce an altitude end condition, the parameters K_1 and K_2 are introduced into Law No. 3:

$$\chi = \tilde{\chi} - K_1 + K_2 t. \tag{11}$$

$$\cos \chi \approx \cos \tilde{\chi} + K_1 \sin \tilde{\chi} - K_2 t \sin \tilde{\chi}$$

$$\sin \chi \approx \sin \tilde{\chi} - K_1 \cos \tilde{\chi} + K_2 t \cos \tilde{\chi} \,. \tag{12}$$

Using these trigonometric expressions in equation (2), the following integrals are found:

$$\dot{y}_T = \dot{y}_1 - gT + \left[\sin \tilde{\chi} - K_1 \cos \tilde{\chi}\right]\left[V_{ex} \ln\left(\frac{\tau}{\tau - T}\right)\right] + K_2 \cos \tilde{\chi}\left[V_{ex}\left(\tau \ln \frac{\tau}{\tau - T} - T\right)\right] \tag{13}$$

$$y_T = y_1 - \frac{1}{2} gT^2 + \dot{y}_1 T + V_{ex}\left[\sin \tilde{\chi} - K_1 \cos \tilde{\chi}\right]\left[T - (\tau - T) \ln\left(\frac{\tau}{\tau - T}\right)\right] + V_{ex} K_2 \cos \tilde{\chi}\left\{-\frac{T^2}{2} + \tau\left[T - (\tau - T) \ln\left(\frac{\tau}{\tau - T}\right)\right]\right\} . \tag{14}$$

Now, equations (8), (9) and (10) enforce the desired velocity end conditions. The introduction of the parameters K_1 and K_2 will perturb the velocity end conditions if K_1 and K_2 are not properly selected. For the orbital injection case, the first order disturbances are reduced in the velocity component normal to the flight path angle at injection. The velocity end condition is preserved by setting the difference of $\dot{y}_T$ and $\dot{\tilde{y}}_T$ to zero,

$$\dot{y}_T - \dot{\tilde{y}}_T = - K \cos \tilde{\chi} V_{ex} \ln\left(\frac{\tau}{\tau - T}\right) + V_{ex} K_2 \cos \tilde{\chi}\left[\tau \ln\left(\frac{\tau}{\tau - T}\right) - T\right] = 0. \tag{15}$$

Equation (14), the altitude end condition, and equation (15), the preserved velocity end condition, can be solved simultaneously for K_1 and K_2. The equations have the form

$$- A_1' K_1 + B_1' K_2 = 0 \tag{16}$$

$$- A_2' K_1 + B_2' K_2 + C_2 = 0, \tag{17}$$

where

$$A_1' = \cos \tilde{\chi} \ln \left(\frac{\tau}{\tau - T} \right) \tag{18}$$

$$B_1' = \cos \tilde{\chi} \left[\tau \ln \frac{\tau}{\tau - T} - T \right] \tag{19}$$

$$A_2' = V_{ex} \cos \tilde{\chi} \left[T - (\tau - T) \ln \left(\frac{\tau}{\tau - T} \right) \right] \tag{20}$$

$$B_2' = V_{ex} \cos \tilde{\chi} \left\{ - \frac{T^2}{2} + \tau \left[T - (\tau - T) \ln \left(\frac{\tau}{\tau - T} \right) \right] \right\} \tag{21}$$

$$C_2' = y_1 - y_T - \frac{1}{2} g T^2 + \dot{y}_1 T + V_{ex} \sin \tilde{\chi}$$

$$\cdot \left[T - (\tau - T) \ln \left(\frac{\tau}{\tau - T} \right) \right] . \tag{22}$$

Therefore,

$$K_1 = \frac{B_1'C_2'}{A_2'B_1' - A_1'B_2'} \tag{23}$$

and

$$K_2 = \frac{A_1'K_1}{B_1'}. \tag{24}$$

Thus, the relation $X = \tilde{X} - K_1 + K_2t$ can be computed stepwise as the current measurements of the state variables are updated. The specified presettings are y_T, $\dot{x}_T$ and $\dot{y}_T$. No enforcement of the terminal range is attempted.

SECTION III. SPHERICAL EARTH WITH GUIDANCE OVER TWO STAGES

The methods employed in the derivation of the guidance equations for a spherical earth are essentially the same as those used with a flat earth model. However, the equations of motion must be modified to account for constantly changing magnitude and direction of the gravity force. Since the gravity force field is conservative and the guidance scheme is a null seeking system, the change in gravity magnitude and direction is approximated. An average gravity magnitude, g^*, and an average gravity direction, ϕ^*, between the current point on the trajectory and the final injection point are updated at the beginning of each pass through the guidance scheme. Thus,

$$R_1^2 = x_1^2 + (R_O + y_1)^2 \tag{25}$$

$$\phi_1 = \tan^{-1}\left(\frac{x_1}{R_O + y_1}\right) \tag{26}$$

$$g_1 = g_O\left(\frac{R_O}{R_1}\right)^2 \tag{27}$$

$$g_T = g_O \left(\frac{R_O}{R_T}\right)^2 \tag{28}$$

$$g^* = \frac{g_1 + g_T}{2} \tag{29}$$

$$\phi^* = \frac{\phi_T - \phi_1}{2}, \tag{30}$$

where the subscript "1" denotes the instantaneous values and the subscript "T" denotes the terminal values. Figure 2 depicts the coordinate system used.

The η axis of the guidance equations' coordinate system is vertical at injection; consequently, the η axis must pass through the injection point (Fig. 2). Therefore, the ξ - η (guidance) coordinate system requires a previous knowledge of ϕ_T, the total range angle. The method employed for deriving the range angle, ϕ_T, will be shown later. The ξ - η coordinate system is formed by rotating the space fixed x - y system through the range angle, ϕ_T(Fig. 2). The x - y system is considered translated to the center of the earth.

Thus,

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} \cos \phi_T & -\sin \phi_T \\ \sin \phi_T & \cos \phi_T \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

and (31)

$$\begin{pmatrix} \dot{\xi} \\ \dot{\eta} \end{pmatrix} = \begin{pmatrix} \cos \phi_T & -\sin \phi_T \\ \sin \phi_T & \cos \phi_T \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$$

The "time-to-go", T, is computed during each guidance cycle along the trajectory as samples of the state variables are taken. The method of computing T without the use of successive approximations will be shown later. The instantaneous velocity deficiency is defined as

$$\Delta V_i^2 = \left(\dot{\xi}_T - \dot{\xi}_1 - g^* T \sin \phi^*\right)^2 + \left(\dot{\eta}_T - \dot{\eta}_1 + g^* T \cos \phi^*\right)^2 . \tag{32}$$

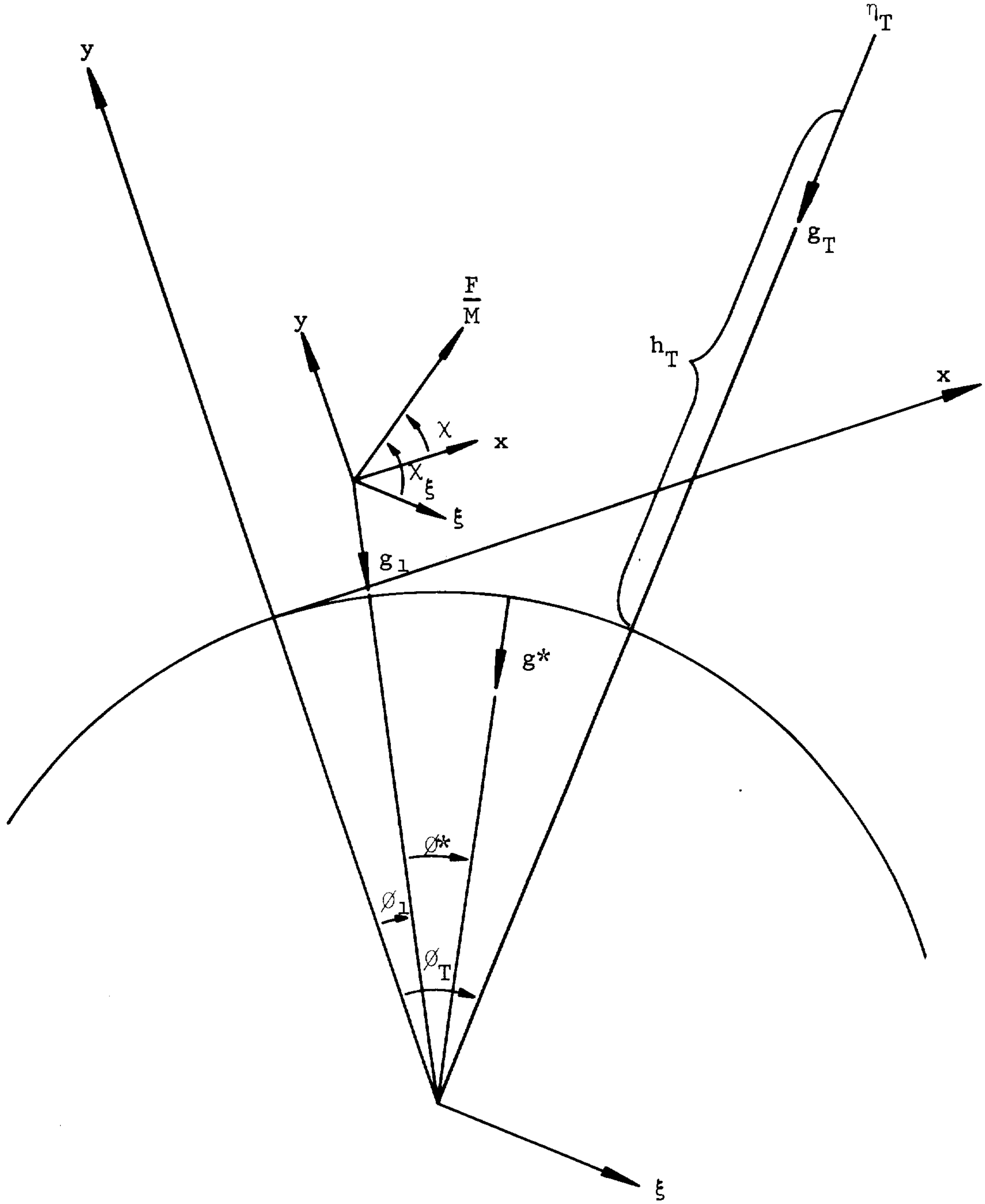

FIGURE 2: FLIGHT GEOMETRY

The constant thrust attitude, referred to the ξ - η coordinate system, necessary to overcome this velocity deficiency at the time T is

$$\tilde{\chi}_{\xi} = \tan^{-1}\left[\frac{\dot{\eta}_T - \dot{\eta}_1 + g^* T \cos \phi^*}{\dot{\xi}_T - \dot{\xi}_1 - g^* T \sin \phi^*}\right]. \tag{33}$$

For two stages of guidance, the thrust attitude angle must be defined as

$$\tilde{\chi}_{\xi} = \tan^{-1}\left[\frac{\dot{\eta}_T - \dot{\eta}_i + g^* (T_1 + T_2) \cos \phi^*}{\dot{\xi}_T - \dot{\xi}_i - g^* (T_1 + T_2) \sin \phi^*}\right], \tag{34}$$

where T_1 is the time-to-go during the first guidance stage and T_2 is the time-to-go during the second guidance stage. T_1 is the time needed to deplete the fuel of the first guidance stage at the present mass flow rate.

Equation (34) enforces the desired cutoff velocity conditions without any altitude constraint. To enforce the terminal altitude condition, it is necessary to introduce the K_1 and K_2 parameters into the steering program. Since the η axis passes through the injection point then the terminal altitude condition is $\eta = \eta_T$, likewise, K_1 and K_2 must be chosen such that $\dot{\eta}_T - \dot{\tilde{\eta}} = 0$. As in the flat earth case the steering program is

$$\chi_{\xi} = \tilde{\chi}_{\xi} - K_1 + K_2 t.$$

It is also required that χ_{ξ} be continuous across staging; therefore, during the second stage of guidance,

$$\chi_{\xi} = \tilde{\chi}_{\xi} - K_1 + K_2 (T_1 + t). \tag{35}$$

The equation of motion in the η direction is

$$\ddot{\eta} = a \sin \tilde{\chi}_{\xi} - g^* \cos \phi^*. \tag{36}$$

Using the trigonometric approximations given in equations (12), the first integral of equation (36) is

$$\dot{\eta}_T = \dot{\eta}_1 - g^* (T_1 + T_2) \cos \phi^* + \left[\sin \tilde{\chi}_\xi - K_1 \cos \tilde{\chi}_\xi\right]\left[V_{ex_1} \ln\left(\frac{\tau_1}{\tau_1 - T_1}\right)\right.$$

$$\left. + V_{ex_2} \ln\left(\frac{\tau_2}{\tau_2 - T_2}\right)\right] + K_2 \cos \tilde{\chi}_\xi \left[V_{ex_2} T_1 \ln\left(\frac{\tau_2}{\tau_2 - T_2}\right)\right.$$

$$\left. + V_{ex_1}\left(\tau_1 \ln \frac{\tau_1}{\tau_1 - T_1} - T_1\right) + V_{ex_2}\left(\tau_2 \ln \frac{\tau_2}{\tau_2 - T_2} - T_2\right)\right], \tag{37}$$

where

V_{ex_1} is the first stage of guidance, thrust over $\dot{m}$,

V_{ex_2} is the second stage of guidance, thrust over $\dot{m}$,

τ_1 is the initial mass over $\dot{m}$ of the first stage of guidance, and

τ_2 is the second stage of guidance mass over $\dot{m}$ of the second stage.

The constant thrust attitude $\tilde{\chi}_\xi$ first integral of equation (36) is

$$\dot{\tilde{\eta}}_T = \tilde{\eta}_1 - g^* (T_1 + T_2) \cos \phi^* + \sin \tilde{\chi}_\xi \left[V_{ex_1} \ln\left(\frac{\tau_1}{\tau_1 - T_1}\right)\right.$$

$$\left. + V_{ex_2} \ln\left(\frac{\tau_2}{\tau_2 - T_2}\right)\right]. \tag{38}$$

The first order perturbations caused by the introduction of K_1 and K_2 must be eliminated; hence, the condition $\dot{\eta}_T - \dot{\tilde{\eta}}_T = 0$ yields

$$- K_1 \cos \tilde{\chi}_\xi \left[V_{ex_1} \ln\left(\frac{\tau_1}{\tau_1 - T_1}\right) + V_{ex_1} \ln\left(\frac{\tau_2}{\tau_2 - T_2}\right)\right] + K_2 \cos \tilde{\chi}_\xi$$

$$\cdot \left[V_{ex_2} T_1 \ln\left(\frac{\tau_2}{\tau_2 - T_2}\right) + V_{ex_1}\left(\tau_1 \ln \frac{\tau_1}{\tau_1 - T_1} - T_1\right)\right.$$

$$\left. + V_{ex_2}\left(\tau_2 \ln \frac{\tau_2}{\tau_2 - T_2} - T_2\right)\right] = 0. \tag{39}$$

The second integral of equation (36) is

$$\eta_T = \eta_1 + \dot{\eta}_1 (T_1 + T_2) - \frac{1}{2} g^* (T_1 + T_2)^2 \cos \phi^* + \left\{\sin \tilde{\chi}_\xi - K_1 \cos \tilde{\chi}_\xi\right\}$$

$$\left\{T_2 V_{ex_1} \ln\left(\frac{\tau_1}{\tau_1 - T_1}\right) + V_{ex_1}\left[T_1 - (\tau_1 - T_1) \ln\left(\frac{\tau_1}{\tau_1 - T_1}\right)\right]\right.$$

$$\left. + V_{ex_2}\left[T_2 - (\tau_2 - T_2) \ln\left(\frac{\tau_2}{\tau_2 - T_2}\right)\right]\right\} + K_2 \cos \tilde{\chi}_\xi$$

$$\cdot\left[T_2 V_{ex_1}\left[\tau_1 \ln\left(\frac{\tau_1}{\tau_1 - T_1}\right) - T_1\right] + T_1 V_{ex_2}\left[T_2 - (\tau_2 - T_2) \ln\left(\frac{\tau_2}{\tau_2 - T_2}\right)\right]\right.$$

$$- V_{ex_1}\left\{\frac{T_1^2}{2} - \tau_1\left[T_1 - (\tau_1 - T_1) \ln\left(\frac{\tau_1}{\tau_1 - T_1}\right)\right]\right\} - V_{ex_2}\left\{\frac{T_2^2}{2}\right.$$

$$\left.\left. - \tau_2\left[T_2 - (\tau_2 - T_2) \ln\left(\frac{\tau_2}{\tau_2 - T_2}\right)\right]\right\}\right]. \tag{40}$$

Equations (39) and (40) solved for K_1 and K_2 yield

$$K_1 = \frac{B_1 C_2}{A_2 B_1 - A_1 B_2} \tag{41}$$

and

$$K_2 = \frac{A_1 K_1}{B_1}, \tag{42}$$

where

$$A_1 = V_{ex_1} \ln\left(\frac{\tau_1}{\tau_1 - T_1}\right) + V_{ex_2} \ln\left(\frac{\tau_2}{\tau_2 - T_2}\right) \tag{43}$$

$$B_1 = V_{ex_1}\left[\tau_1 \ln\left(\frac{\tau_1}{\tau_1 - T_1}\right) - T_1\right] + V_{ex_2}\left[T_1 \ln\left(\frac{\tau_2}{\tau_2 - T_2}\right)\right.$$

$$\left. + \tau_2 \ln\left(\frac{\tau_2}{\tau_2 - T_2}\right) - T_2\right] \tag{44}$$

$$A_2 = \cos\tilde{\chi}_\xi \left\{T_2 V_{ex_1} \ln\left(\frac{\tau_1}{\tau_1 - T_1}\right) + V_{ex_1}\left[T_1 - (\tau_1 - T_1)\ln\left(\frac{\tau_1}{\tau_1 - T_1}\right)\right]\right.$$

$$\left. + V_{ex_2}\left[T_2 - (\tau_2 - T_2)\ln\left(\frac{\tau_2}{\tau_2 - T_2}\right)\right]\right\} \tag{45}$$

$$B_2 = \cos\tilde{\chi}_\xi \left[T_2 V_{ex_1}\left[\tau_1 \ln\left(\frac{\tau_1}{\tau_1 - T_1}\right) - T_1\right] + T_1 V_{ex_2}\left[T_2\right.\right.$$

$$\left. - (\tau_2 - T_2)\ln\left(\frac{\tau_2}{\tau_2 - T_2}\right)\right] - V_{ex_1}\left\{\frac{T_1^2}{2} - \tau_1\left[T_1 - (\tau_1 - T_1)\ln\left(\frac{\tau_1}{\tau_1 - T_1}\right)\right]\right\}$$

$$\left. - V_{ex_2}\left\{\frac{T_2^2}{2} - \tau_2\left[T_2 - (\tau_2 - T_2)\ln\left(\frac{\tau_2}{\tau_2 - T_2}\right)\right]\right\}\right] \tag{46}$$

$$C_2 = \eta_1 - \eta_T + \dot{\eta}_1 (T_1 + T_2) - \frac{1}{2} g^* (T_1 + T_2)^2 \cos\phi^*$$

$$+ \sin\tilde{\chi}_\xi\left\{V_{ex_1}\left[T_2 \ln\left(\frac{\tau_1}{\tau_1 - T_1}\right) + T_1 - (\tau_1 - T_1)\ln\left(\frac{\tau_1}{\tau_1 - T_1}\right)\right]\right.$$

$$\left. + V_{ex_2}\left[T_2 - (\tau_2 - T_2)\ln\left(\frac{\tau_2}{\tau_2 - T_2}\right)\right]\right\} . \tag{47}$$

For explanation purposes, a coast period has not been included. The coast period does not modify the form of the equations; however, the coefficients of equations (39) and (40) would have some additional terms.

Whenever staging occurs, T_1 is set to zero; thus, no modification of the steering program equations is required. Since staging has been previously accounted for, the guidance computation proceeds smoothly. The problem is finding a proper method of computing ϕ_T and T_2.

SECTION IV. THE COMPUTATION OF $\emptyset_T$ AND T_2

The total range $\emptyset_T$ may be either preset at launch or it may be computed in flight. The preset $\emptyset_T$ works very well except for cases where the actual range angle exceeds the presetting. If the preset is too small, the trajectory tends to be too steep, causing a loss in performance. When the range angle exceeds the preset $\emptyset_T$, the sign of $\emptyset^*$ reverses since the injection point no longer lies on the η axis. An instability may set up in the steering program if this condition prevails. To overcome this difficulty, it is necessary to step $\emptyset_T$ forward as the angle approaches $\emptyset_T$. If a preset $\emptyset_T$ is used, some of the adaptivity of the scheme is lost for relatively large disturbances.

The more general cases are covered by computing the total range angle in flight. The approximations used in this particular method contain small errors during the initial portion of guidance; however, these errors reduce significantly as the flight progresses. The range angle is computed as follows:

The distance that would be covered by a horizontal flight over a flat earth during the first stage of guidance is

$$X_1 = V_1 T_1 + V_{ex_1} \left[T_1 - (\tau_1 - T_1) \ln \left(\frac{\tau_1}{\tau_1 - T_1} \right) \right] . \tag{48}$$

Dividing by the terminal radius gives an approximation for the first stage range angle,

$$\emptyset_{11} = \frac{1}{R_T} \left\{ V_1 T_1 + V_{ex_1} \left[T_1 - (\tau_1 - T_1) \ln \left(\frac{\tau_1}{\tau_1 - T_1} \right) \right] \right\} . \tag{49}$$

Using equation (49), the second stage velocity deficiency is computed by

$$\Delta V_i = V_1 + V_{ex_1} \ln \left(\frac{\tau_1}{\tau_1 - T_1} \right) - V_T - g^* T_1 \sin \frac{\emptyset_{11}}{2}, \tag{50}$$

where V_T is the preset cutoff velocity. The second stage characteristic velocity equation is

$$\Delta V = V_{ex_2} \ln \left(\frac{\tau_2}{\tau_2 - T_2^*} \right) , \tag{51}$$

where T_2^* is the second stage estimated time-to-go.

Equations (50) and (51) yield

$$T_2^* = \tau_2 \left\{ 1 - \mathrm{EXP} \left[\frac{V_1 + V_{ex_1} \ln\left(\dfrac{\tau_1}{\tau_1 - T_1}\right) - V_T - g^* \ T_1 \sin\left(\dfrac{\phi_{11}}{2}\right)}{V_{ex_2}} \right] \right\} . \tag{52}$$

The estimated second stage range angle is computed by

$$\phi_{12} = \frac{1}{R_T} \left\{ \left[V_1 + V_{ex_1} \ln\left(\frac{\tau_1}{\tau_1 - T_1}\right) - g^* \ T_1 \sin\left(\frac{\phi_{11}}{2}\right) \right] T_2^* \right.$$

$$\left. + V_{ex_2} \left[T_2^* - (\tau_2 - T_2^*) \ln\left(\frac{\tau_2}{\tau_2 - T_2}\right) \right] \right\} . \tag{53}$$

Then the total range angle is

$$\phi_T = \phi_1 + \phi_{11} + \phi_{12} , \tag{54}$$

and the average gravity direction is

$$\phi^* = \frac{1}{2} (\phi_{11} + \phi_{12}) , \tag{55}$$

where ϕ_1 is the instantaneous range angle.

Since ϕ_{11} has no gravity losses taken into account, ϕ_{11} will at first be too large. However, this error quickly diminishes as the burn time decreases. The error in ϕ_{11} causes ϕ_{12} to be too small. The overall effect is to reduce the total error so that the sum of ϕ_{11} and ϕ_{12} produces a surprisingly good estimate of the total range angle ϕ_T. Since the velocity of the vehicle is continually increasing, the larger portion of the flight time takes place in the lower half of the total range angle. For particular missions with long burn times, this approximation tends to be inaccurate; hence, it is necessary to use a weighted average for ϕ^* and g^*.

The computation of T_2, without some method of successive approximations, requires some knowledge of the length of the second stage time-to-go. Either an initial estimate may be preset or the estimated burn time from equation (52) may be used. After the first pass through the guidance package, the newly computed T_2 is used for the next pass. After the first stage burnout and separation occur, the cycle time of the guidance package is subtracted from the old T_2 and this value used for each succeeding pass.

Let δT_2 be the error in T_2, the time-to-go, and let T_2' be the estimated time-to-go; then,

$$T_2 = T_2' + \delta T_2. \tag{56}$$

The velocity deficiency equation may be written as

$$\Delta V_1 = \left\{ \left[\dot{\xi}_T - \dot{\xi}_1 - g^* \, (T_1 + T_2' + \delta T_2) \sin \phi^* \right]^2 + \left[\dot{\eta}_T - \dot{\eta}_1 + g^* \, (T_1 + T_2' + \delta T_2) \cos \phi^* \right]^2 \right\}^{\frac{1}{2}}. \tag{57}$$

Let

$$\Delta \dot{\xi} = \dot{\xi}_T - \dot{\xi}_1 - g^* \, (T_1 + T_2') \sin \phi^*, \tag{58}$$

$$\Delta \dot{\eta} = \dot{\eta}_T - \dot{\eta}_1 + g^* \, (T_1 + T_2') \cos \phi^*, \tag{59}$$

and

$$\lambda = g^* \left[\Delta \dot{\eta}^* \cos \phi^* - \Delta \dot{\xi}^* \sin \phi^* \right] \tag{60}$$

$$(\Delta V^*)^2 = (\Delta \dot{\xi}^*)^2 + (\Delta \dot{\eta}^*)^2. \tag{61}$$

Then

$$(\Delta V_1)^2 = \left[\Delta \dot{\xi}^* - g^* \, (\delta T_2) \sin \phi^* \right]^2 + \left[\Delta \dot{\eta}^* + g^* \, (\delta T_2) \cos \phi^* \right]^2, \tag{62}$$

or

$$(\Delta V_1)^2 = (g^*)^2 \, (\delta T_2)^2 + 2\lambda \, (\delta T_2) + (\Delta V^*)^2. \tag{63}$$

The characteristic velocity equation is

$$\Delta V = V_{ex_1} \ln \left(\frac{\tau_1}{\tau_1 - T_1} \right) + V_{ex_2} \ln \left[\frac{\tau_2}{\tau_2 - (T_2' + \delta T_2)} \right]. \tag{64}$$

But

$$\ln\left[\frac{\tau_2}{\tau_2 - (T_2' + \delta T_2)}\right] = \ln\left(\frac{\tau_2}{\tau_2 - T_2'}\right) - \ln\left(1 - \frac{\delta\tau_2}{\tau_2 - T_2'}\right) . \tag{65}$$

Now

$$\ln\left(1 - \frac{\delta T_2}{\tau_2 - T_2'}\right) = \frac{-\delta T_2}{T_2 - T_2'} - \frac{1}{2}\left(\frac{-\delta T_2}{\tau_2 - T_2'}\right)^2 + \ldots \tag{66}$$

Hence, equation (64) can be written as

$$\Delta V = V_{ex_1} \ln\left(\frac{\tau_1}{\tau_1 - T_1}\right) + V_{ex_2} \ln\left(\frac{\tau_2}{\tau_2 - T_2'}\right) - V_{ex_2}\left[\frac{-\delta T_2}{\tau_2 - T_2'} - \frac{1}{2}\left(\frac{\delta T_2}{\tau_2 - T_2'}\right)^2 + \ldots\right] \tag{67}$$

Assume that T_2' is a reasonably good estimate of the second stage burn time; then the terms $1/2\ (-\delta T_2/\tau_2 - T_2')^2$ and higher order are small and may be neglected.

Let

$$L = V_{ex_1} \ln\left(\frac{\tau_1}{\tau_1 - T_1}\right) + V_{ex_2} \ln\left(\frac{\tau_2}{\tau_2 - T_2'}\right) , \tag{68}$$

and

$$K = \frac{V_{ex_2}}{\tau_2 - T_2'} . \tag{69}$$

Then, by substituting equations (68) and (69) into equation (67) and squaring, δT_2 can be solved from equations (63) and (67),

$$(g^*)^2\ (\delta T_2)^2 + 2\lambda\ (\delta T_2) + (\Delta V^*) = K^2\ (\delta T_2)^2 + 2LK\ (\delta T_2) + L^2. \tag{70}$$

Let

$$a = K^2 - (g^*)^2 , \tag{71}$$

$$b = \lambda - LK, \tag{72}$$

$$c = (\Delta V^*)^2 - L^2 . \tag{73}$$

Then equation (70) can be written as

$$-a\ (\delta T_2)^2 + 2b\ (\delta T_2) + c = 0. \tag{74}$$

Therefore,

$$\delta T_2 = \frac{b + \sqrt{b^2 + ac}}{a} . \tag{75}$$

Since the coefficient b is negative, the positive radical is chosen in equation (75) to produce the smaller δT_2.

Using $T_2 = T_2' + \delta T_2$ and equation (75) for $\emptyset^*$, $\tilde{X}_\xi$ is computed by

$$\tilde{X}_\xi = \tan^{-1} \left[\frac{\Delta\dot{\eta}^* + g^* (\delta T_2) \cos \emptyset^*}{\Delta\dot{\xi}^* - g^* (\delta T_2) \sin \emptyset} \right] . \tag{76}$$

The indeterminate nature of equation (76), mentioned in the description, as T_2 approaches zero can be handled in two ways. The generally used method is to freeze all the guidance parameters, $\tilde{X}_\xi$, K_1, K_2 and T_2 as $T_2 < \epsilon$, where ϵ is some arbitrarily small time-to-go. The data presented at the end of the paper was computed using this method. It has been found that the constant ϵ is not critical; in fact, an ϵ of up to 20 seconds causes negligible dispersions in the desired terminal conditions.

The second method is to determine the rate of change of the velocity components and use the limit equation

$$\tilde{X}_\xi = \underset{T_2 \to 0}{\mathrm{LIM}} \left[\tan^{-1} \left(\frac{-\Delta\ddot{\eta} T_2 + g^* \cos \emptyset^*}{-\Delta\ddot{\xi} T_2 - g^* \sin \emptyset^*} \right) \right] \tag{77}$$

as prescribed by L'Hospital's Rule. It has been found that this method does not appreciably improve the end conditions over the $T_2 < \epsilon$ method.

The steering equation now derived is referenced to the horizontal at injection. It is necessary then to rotate the steering function back into the space fixed system; hence,

$$\chi = \tilde{X}_{\xi} - K_1 - \phi_T + K_2 \, (t - t_O), \tag{78}$$

where t_O is the instant in running when χ was last computed, and t is the running time.

SECTION V. IMPLEMENTATION PROCEDURE

The order in which the guidance equations were derived does not determine the order of computation. Before computing the steering function the total range and the suceeding burning time of each stage must be known. Before computing T_2, the state variables must be rotated into the $\xi - \eta$ system of which one axis passes through the injection point. T_2 must now be computed before finding $\tilde{X}_{\xi}$. Once $\tilde{X}_{\xi}$ and T_2 are determined, the coefficients of K_1 and K_2 can be evaluated. After K_1 and K_2 are computed, the steering program must then be referenced to the inertial coordinate system. A flow diagram will demonstrate the proper computing sequence. Figure 3 depicts the flow of two-stage guidance equations in the guidance computer.

Although two stages of guidance are shown, the equations can be transformed into one stage simply by setting $T_1 = 0$ and τ_1 to some arbitrary constant. If some engine-out capability is desired, then at the instant of the engine failure, the guidance scheme is reinitialized by adjusting either τ_1 and V_{ex_1} or τ_2 and V_{ex_2} depending on which stage is in operation. This reinitialization is a rapid process and has no noticeable effect on the overall performance. Engine-out capability is not included in the flow diagram.

The equations needed to compute the guidance constants are

$$V_1^2 = \dot{x}_1^2 + \dot{y}^2 \tag{79}$$

$$R_1^2 = x_1^2 + y_1^2 \tag{25}$$

where the reference system is considered translated to the center of the earth.

$$\phi_1 = \tan^{-1}\left(\frac{x_1}{y_1}\right) \tag{26}$$

$$g_1 = g_O \left(\frac{R_O}{R_1}\right)^2 \tag{27}$$

$$g_T = g_0 \left(\frac{R_0}{R_T}\right)^2 \tag{28}$$

Where R_T^2 is a presetting.

$$g^* = \frac{g_1 + g_T}{2} \tag{29}$$

$$\phi_{11} = \frac{1}{R_T} \left\{ V_1 T_1 + V_{ex_1} \left[T_1 - (\tau_1 - T_1) \ln \left(\frac{\tau_1}{\tau_1 - T_1} \right) \right] \right\} \tag{49}$$

$$T_2^* = \tau_2 \left\{ 1 - \text{EXP} \left[\frac{V_1 + V_{ex_1} \ln \left(\frac{\tau_1}{\tau_1 - T_1} \right) - V_T - g^* T_1 \sin \left(\frac{\phi_{11}}{2} \right)}{V_{ex_2}} \right] \right\} \tag{52}$$

$$\phi_{12} = \frac{1}{R_T} \left\{ \left[V_1 + V_{ex_1} \ln \left(\frac{\tau_1}{\tau_1 - T_1} \right) - g^* T_1 \sin \left(\frac{\phi_{11}}{2} \right) \right] T_2^* \right.$$

$$\left. + V_{ex_2} \left[T_2^* - (\tau_2 - T_2^*) \ln \left(\frac{\tau_2}{\tau_2 - T_2^*} \right) \right] \right\} \tag{53}$$

$$\phi_T = \phi_1 + \phi_{11} + \phi_{12} \tag{54}$$

$$\phi^* = \frac{1}{2} (\phi_{11} + \phi_{12}) \tag{55}$$

$$\begin{pmatrix} \xi_1 \\ \eta_1 \end{pmatrix} = \begin{pmatrix} \cos \phi_T & -\sin \phi_T \\ \sin \phi_T & \cos \phi_T \end{pmatrix} \begin{pmatrix} X_1 \\ Y_1 \end{pmatrix}$$

$$\begin{pmatrix} \dot{\eta}_1 \\ \dot{\xi}_1 \end{pmatrix} = \begin{pmatrix} \cos \phi_T & -\sin \phi_T \\ \sin \phi_T & \cos \phi_T \end{pmatrix} \begin{pmatrix} \dot{X}_1 \\ \dot{Y}_1 \end{pmatrix} \tag{31}$$

$$\Delta \dot{\xi}^* = \dot{\xi}_T - \dot{\xi}_1 - g^* (T_1 + T_2') \sin \phi^* \tag{58}$$

$$\Delta \dot{\eta}^* = \dot{\eta}_T - \dot{\eta}_1 + g^* (T_1 + T_2') \cos \phi^* \tag{59}$$

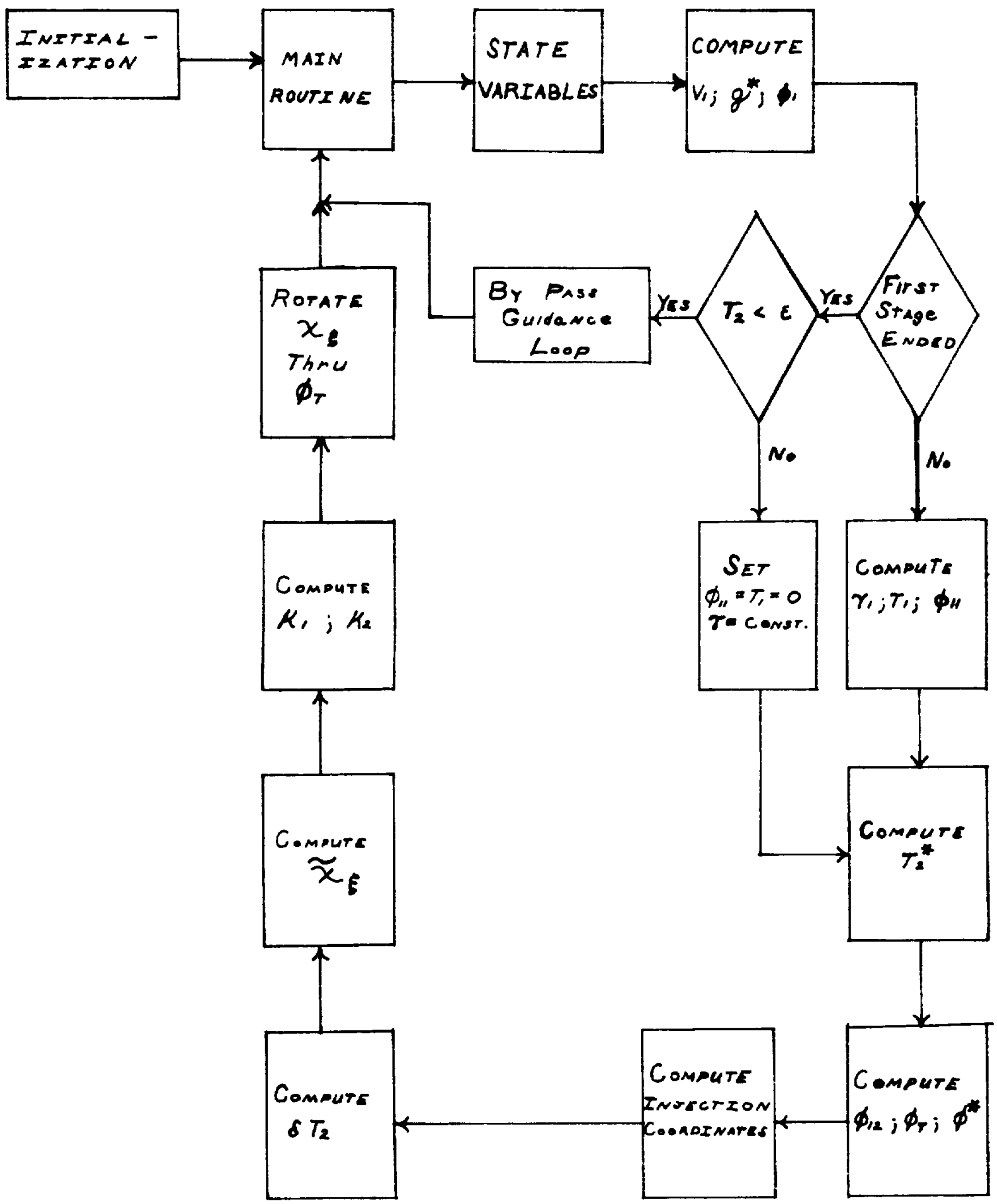

FIGURE 3: FLOW DIAGRAM

where $\dot{\xi}_T$ and $\dot{\eta}_T$ are preset values determined by the desired terminal path angle.

$$\lambda = g^* \left[\Delta\dot{\eta}^* \cos \phi^* - \Delta\dot{\xi}^* \sin \phi^*\right] \tag{60}$$

$$(\Delta V^*)^2 = (\Delta\dot{\eta}^*)^2 + (\Delta\dot{\xi}^*)^2 \tag{61}$$

$$L = V_{ex_1} \ln \left(\frac{\tau_1}{\tau_1 - T_1}\right) + V_{ex_2} \ln \left(\frac{\tau_2}{\tau_2 - T_2'}\right) \tag{68}$$

$$K = \frac{V_{ex_2}}{\tau_2 - T_2'} \tag{69}$$

$$a = K^2 - (g^*)^2 \tag{71}$$

$$b = \lambda - LK \tag{72}$$

$$c = (\Delta V^*)^2 - L^2 \tag{73}$$

$$\delta T_2 = \frac{b + \sqrt{b^2 + ac}}{a} \tag{75}$$

$$T_2 = T_2' + \delta T_2 \tag{56}$$

$$\tilde{\chi}_\xi = \tan^{-1} \left[\frac{\Delta\dot{\eta}^* + g^* (\delta T_2) \cos \phi^*}{\Delta\dot{\xi}^* - g^* (\delta T_2) \sin \phi^*}\right] \tag{76}$$

$$A_1 = V_{ex_1} \ln \left(\frac{\tau_1}{\tau_1 - T_1}\right) + V_{ex_2} \ln \left(\frac{\tau_2}{\tau_2 - T_2}\right) \tag{43}$$

$$B_1 = V_{ex_1} \left[\tau_1 \ln \left(\frac{\tau_1}{\tau_1 - T_1}\right) - T_1\right] + V_{ex_2} \left[T_1 \ln \left(\frac{\tau_2}{\tau_2 - T_2}\right) + \tau_2 \ln \left(\frac{\tau_2}{\tau_2 - T_2}\right) - T_2\right] \tag{44}$$

$$A_2 = \cos \tilde{X}_\xi \left\{ T_2 V_{ex_1} \ln \left(\frac{\tau_1}{\tau_1 - T_1} \right) + V_{ex_1} \left[T_1 - (\tau_1 - T_1) \ln \left(\frac{\tau_1}{\tau_1 - T_1} \right) \right] \right.$$

$$\left. + V_{ex_2} \left[T_2 - (\tau_2 - T_2) \ln \left(\frac{\tau_2}{\tau_2 - T_2} \right) \right] \right\} \tag{45}$$

$$B_2 = \cos \tilde{X}_\xi \Bigg[T_2 V_{ex_1} \left[\tau_1 \ln \left(\frac{\tau_1}{\tau_1 - T_1} \right) - T_1 \right] + T_1 V_{ex_2} \left[T_2 \right.$$

$$\left. - (\tau_2 - T_2) \ln \left(\frac{\tau_2}{\tau_2 - T_2} \right) \right] - V_{ex_1} \left\{ \frac{T_1^2}{2} - \tau_1 \left[T_1 - (\tau_1 - T_1) \ln \left(\frac{\tau_1}{\tau_1 - T_1} \right) \right] \right\}$$

$$- V_{ex_2} \left\{ \frac{T_2^2}{2} - \tau_2 \left[T_2 - (\tau_2 - T_2) \ln \left(-\frac{\tau_2}{\tau_2 - T_2} \right) \right] \right\} \Bigg] \tag{46}$$

$$C_2 = \eta_1 - \eta_T + \dot{\eta}_1 (T_1 + T_2) - \frac{1}{2} g^* (T_1 + T_2)^2 \cos \phi^*$$

$$+ \sin \tilde{X}_\xi \left\{ V_{ex_1} \left[T_2 \ln \left(\frac{\tau_1}{\tau_1 - T_1} \right) + T_1 - (\tau_1 - T_1) \ln \left(\frac{\tau_1}{\tau_1 - T_2} \right) \right] \right.$$

$$\left. + V_{ex_2} \left[T_2 - (\tau_2 - T_2) \ln \left(\frac{\tau_2}{\tau_2 - T_2} \right) \right] \right\} \tag{47}$$

$$K_1 = \frac{B_1 C_2}{A_2 B_1 - A_1 B_2} \tag{41}$$

$$K_2 = \frac{A_1 K_1}{B_1} \, . \tag{42}$$

The equations are listed in the form as they appear during the derivation. An inspection of this set will reveal that it is possible to reduce the computation by combinations. However, the purpose of this report is to demonstrate the principles involved in the guidance scheme and not to present the scheme in its final form.

SECTION VI. NUMERICAL INVESTIGATION

The Saturn V vehicle, suborbital start of the third stage, was used as a model to demonstrate the adaptive nature of the guidance system presented. However, no data or characteristics of the Saturn V

vehicle itself are presented. The tables are all in terms of Delta performance and percentages. A realistic comparison was desired; hence, for each case presented a calculus of variations program was run for comparison data. This means that the data is not a comparison to some nominal standard trajectory, but a comparison to what the calculus of variations would have done if faced with the same situation. The guidance loop was closed after the Saturn V first stage burnout and coast period. At this point dispersions were introduced to generate a family of orbital injection trajectories.

The weakest link in the guidance system is the average gravity magnitude and direction approximations. For single stages with relatively short burn times and/or relatively steep trajectories, the approximations are excellent. However, as the trajectories become flatter with longer burn times, then the larger total range angles cause the approximation to begin to breakdown. This is not serious if it is known that the missile characteristics will produce such a trajectory. It is only necessary to modify the g^* equation (29) and the ϕ^* equation (55) to a set of weighted average equations. It is even feasible to consider a time varying weighted average; however, this is not done in this report.

To demonstrate the weighted average effects, two cases are presented:

$$\text{Case A,} \qquad \phi^* = \frac{1}{2} (\phi_{11} + \phi_{12}) \ , \tag{55}$$

$$\text{Case B,} \qquad \phi^* = \frac{8}{17} (\phi_{11} + \phi_{12}). \tag{80}$$

However, no cases are presented with a weighted average gravity magnitude g^*. Each case was run on a typical Saturn V orbital injection trajectory with seven large perturbations introduced to form the family of trajectories. The results are presented in Table 1. All cases were run with the guidance output freezing at $T_2 < \epsilon$ method. For this study, an ϵ of ten seconds was used. The terminal conditions were so close to the desired end conditions that it was not considered necessary to look at any smaller ϵ. The results are presented in Table 2. Since it was assumed that the vehicle had a perfect autopilot, the errors presented are strictly scheme errors. A study of the data presented will show that the scheme controls the terminal conditions very tightly with very little loss in performance.

Changing to a weighted average gravity direction causes a change in the trajectory shape. Case A mentioned above is an altitude overshoot trajectory with a subsequent performance loss. Although no attempt was made to strictly optimize the weighted average, Case B is more nearly an optimum trajectory. Figure 4 presents an altitude versus velocity plot of Case A, Case B, and the calculus of variations standard case. Some trajectory shaping could be accomplished merely by varying the weighted average equations (55) and (29).

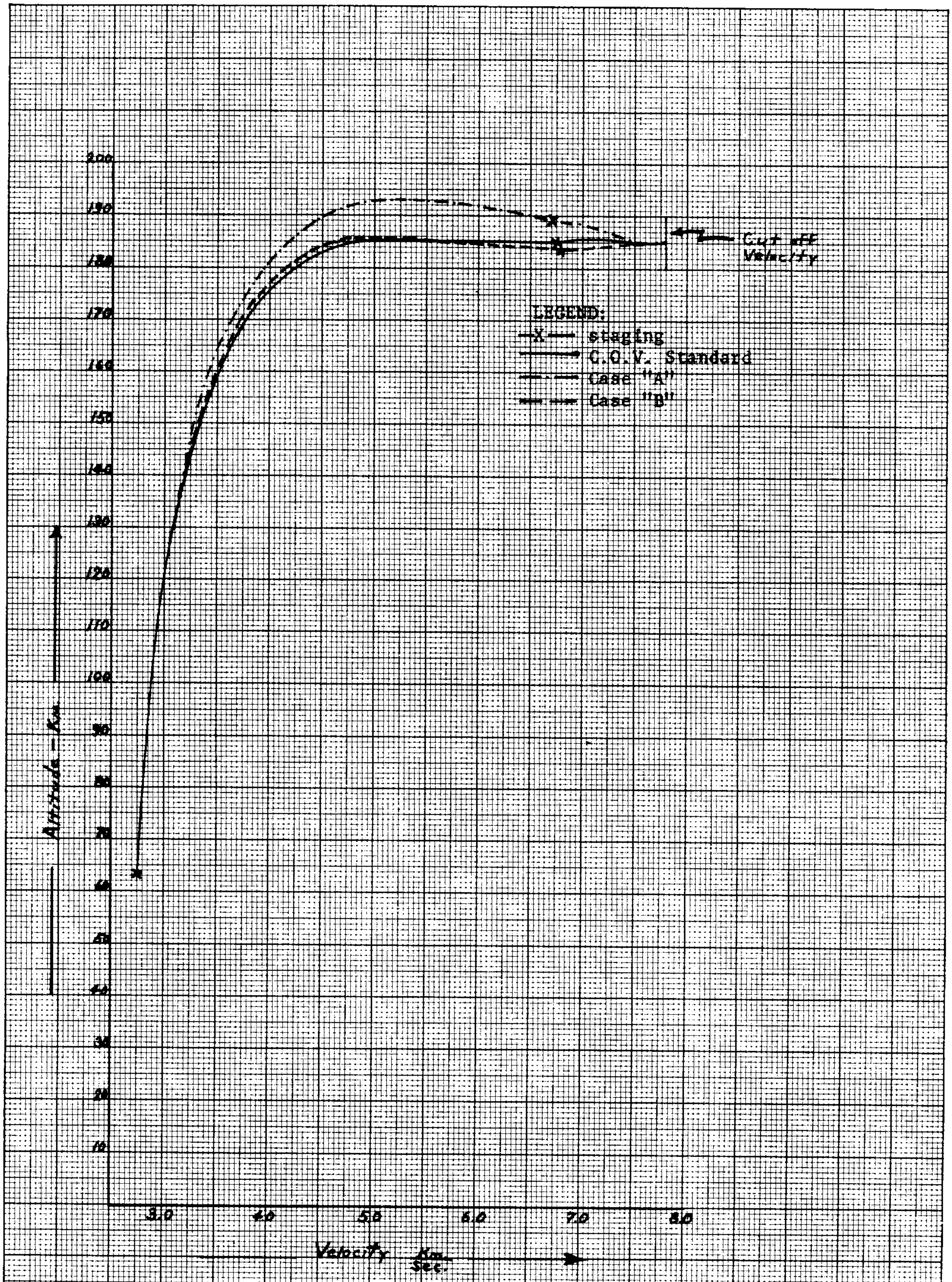

FIGURE 4: ALT. VS. VELOCITY TRAJECTORY REACTION TO β^* COMPUTATION

A numerical study was also conducted using an oblate earth model in the differential equations of motion. The guidance scheme itself still used spherical earth gravity approximations to determine the steering coefficients and the time-to-go. This particular study did not use the weighted average gravity magnitude and direction mentioned earlier in this section. Since a seventy degree azimuth was used, the oblate earth altitude is no longer 185.2 km for a hundred nautical mile orbit referenced to Cape Canaveral. The first stage was flown with a time polynomial tilt program with guidance inserted at the ignition of the second stage. Table 3 shows the results of this study. The data was generated and prepared by the Boeing Company under contract for the Aeroballistics Division of the George C. Marshall Space Flight Center.

SECTION VII. CONCLUSIONS

The guidance system outlined in this report is another approach to the path adaptive guidance mechanization problem. Once the relatively few presettings have been determined, the scheme is capable of handling large booster stage disturbances. The scheme requires some subroutines like natural logarithm, arc tangent, sine and cosine, square root, etc.

The guidance output is an initial thrust attitude, a rate of turn of that thrust attitude, and a time left to cutoff. At discrete intervals of time the state variables sample is updated and a new guidance output is generated. Since a turning rate is generated, the discontinuities that normally occur in polynomial steering between steps is minimized. An indeterminate function in the steering formula occurs if the guidance parameters are evaluated very near cutoff. This difficulty is eliminated by either applying L'Hospital's rule of limits or by freezing the steering constants at some arbitrary time-to-go. The indeterminancy is not serious as long as it is accounted for, since it does not affect the terminal conditions.

The weakest link in the system is the average gravity direction and magnitude computation. However, if it is known what type of trajectory the vehicle is required to fly, then a proper weighting function can be found that produces an optimum trajectory.

TABLE 1

PERFORMANCE LOSS OF THE SCHEME IN PERCENT OF C.O.V. MASS IN ORBIT

	Initial State Variables				Performance Loss	
Case	x_1	$\dot{x}_1$	y_1	$\dot{y}_1$	Percent from Optimum	
Number	Km	m/sec	Km	m/sec	Case A	Case B
1	151.28	2556.6	63.974	930.53	0.11%	0.04%
2	154.00	2556.6	63.974	930.53	0.11%	0.04%
3	151.28	2700.0	63.974	930.53	0.32%	0.09%
4	151.28	2500.0	63.974	930.53	0.07%	0.05%
5	151.28	2556.6	65.000	930.53	0.11%	0.04%
6	151.28	2556.6	62.000	930.53	0.11%	0.04%
7	151.28	2556.6	63.974	1000.00	0.14%	0.05%
8	151.28	2556.6	63.974	850.00	0.11%	0.03%

TABLE II

TERMINAL CONDITIONS AT CUTOFF OF THE GUIDANCE SCHEME

DESIRED CONDITIONS ALT. = 185.200 kilometers
Velocity = 7794.7 m/sec
Path Angle = 0.0 degrees from the local horizontal

Case Number	Case A			Case B		
	Altitude	Velocity	Path Angle	Altitude	Velocity	Path Angle
	Kilometers	m/sec	Deg. from Horizontal	Kilometers	m/sec	Deg. from Horizontal
1	185.19995	7794.679	+.000909	185.20000	7794.714	+.000275
2	185.19991	7794.680	+.000890	185.19999	7794.713	+.000268
3	185.19987	7794.642	+.000327	185.20001	7794.707	+.000165
4	185.19997	7794.686	+.000878	185.19999	7794.715	+.000424
5	185.19998	7794.680	+.000797	185.19998	7794.713	+.000246
6	185.19997	7794.679	+.001130	185.19999	7794.716	+.002540
7	185.19992	7794.675	+.000191	185.19999	7794.707	+.002521
8	185.19998	7794.695	+.000603	185.19999	7794.709	+.000914

TABLE 3

OBLATE EARTH MODEL PERTURBATION EFFECTS ON THE GUIDANCE END CONDITIONS

Perturbation	Percent Deviation	Resulting End Conditions			
		Alt. Km	Velocity M/sec	Path Angle Degrees	Range N. Miles
Baseline Guidance	0	187.58	7795.40	89.987	1425.0
S-IC Thrust	- 3	187.61	7795.39	89.988	1454.1
S-IC Thrust	+ 3	187.58	7795.39	89.987	1414.6
S-II Thrust	- 3	187.62	7795.38	89.988	1462.4
S-II Thrust	+ 3	187.55	7795.41	89.985	1390.9
S-IVB Thrust	- 3	187.60	7792.90	89.989	1443.2
S-IVB Thrust	+ 3	187.57	7797.90	89.982	1408.1
S-IC I_{sp}	- 1	187.59	7795.40	89.987	1436.4
S-IC I_{sp}	+ 1	187.57	7795.40	89.986	1413.6
S-II I_{sp}	- 1	187.59	7795.40	89.987	1434.6
S-II I_{sp}	+ 1	187.58	7795.40	89.987	1415.4
S-IVB I_{sp}	- 1	187.58	7795.57	89.986	1424.4
S-IVB I_{sp}	+ 1	187.58	7795.24	89.987	1425.6
S-IC Wt. Uncertainty	-.3	187.58	7795.40	89.987	1424.2
S-IC Wt. Uncertainty	+.3	187.59	7795.40	89.987	1425.9

TABLE 3 (Cont'd)

Perturbation	Percent Deviation	Resulting End Conditions Alt. Km	Velocity M/Sec	Path Angle Degrees	Range N. Miles
S-II Wt. Uncertainty	-.3	187.58	7795.40	89.987	1424.2
S-II Wt. Uncertainty	+.3	187.59	7795.40	89.987	1425.8
S-IVB Wt. Uncertainty	-.3	187.58	7795.40	89.987	1424.9
S-IVB Wt. Uncertainty	+.3	187.58	7795.40	89.987	1425.1
Air Density	- 3	187.58	7795.40	89.987	1424.4
Air Density	+ 3	187.58	7795.40	89.987	1424.4
Drag	-10	187.58	7795.41	89.987	1422.2
Drag	+10	187.59	7795.40	89.987	1428.2

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